1.3 Algebraic Limits

Obj: Define limits and the properties of limits; Evaluate limits graphically and algebraically

<u>Finding limits Analytically.</u>

1.

2.

3.

Properties of limits.

$\lim_{x \to c}(f(x)+g(x)) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$	$\lim_{x \to c}(f(x)-g(x)) = \lim_{x \to c} f(x) - \lim_{x \to c} g(x)$
$\lim_{x \to c}(f(x) \bullet g(x)) = \lim_{x \to c} f(x) \bullet \lim_{x \to c} g(x)$	$\lim_{x \to c} k(f(x)) = k \lim_{x \to c} f(x)$
$\lim_{x \to c}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$	$\lim_{x \to c}(f(x))^n = (\lim_{x \to c} f(x))^n$

Solving algebraically examples. **<u>ALWAYS TRY DIRECT SUBSTITUTION</u>**

1. $\lim_{x \to -1} 2x^2 + 3x =$

2. $\lim_{x \to 1} \frac{2x^2 - 1}{x + 1} =$

3. $\lim_{x \to 0} \frac{e^x - \tan x}{\cos^2 x} =$

Now graph the following limits and predict #3.

1. $\lim_{x \to 0} \frac{\sin x}{x} =$

2. $\lim_{x \to 0} \frac{\cos x}{x} =$

3. $\lim_{x \to 0} \frac{\tan x}{x} =$

When substitution results in a c/0 fraction, where c is some non zero number, the function has a vertical asymptote and the limit is only one sided. The overall limit is DNE!

Example. $\lim_{x\to 4}\frac{x+2}{x-4}=$

When substitution results in a 0/0 fraction, the result is called an indeterminate form. You must solve this limit another way (factoring, conjugates, trig identities, clear fractions

Example. $\lim_{x\to -2}\frac{x+2}{x^2-4}=$

What about: $\lim_{x\to 2}\frac{x+2}{x^2-4}=$

You try.

$\lim_{x\to 1}\frac{x^3-1}{x-1}=$

$\lim_{x\to -3}\frac{x^2+x-6}{x+3}=$

<u>Limits with roots.</u>

$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=$

$\lim_{x\to 36}\frac{\sqrt{x}-6}{x-36}=$

Absolute value: Separate into a piecewise functions

$\lim_{x\to 3}\frac{|x-3|}{x-3}=$

Use your properties of limits.

Given $\lim_{x\to 5} f(x) = 3$ and $\lim_{x\to 5} g(x) = 2$

What would you expect these to be?

$\lim_{x\to 5}\left[f(x)+g(x)\right]$ $\lim_{x\to 5}\left[f(x)-g(x)\right]$ $\lim_{x\to 5}\left[f(x)\bullet g(x)\right]$

$\lim_{x\to 5}\left[2f^2(x)-3f(x)g(x)\right]$

Use this limit $\lim_{x\to 0}\frac{\sin x}{x}=1$ to help solve these limits.

1. $\lim_{x\to 0}\frac{x+\sin x}{x}=$ 2. $\lim_{x\to 0}\frac{1-\cos^2 x}{x^2}=$ 3. $\lim_{x\to 0}\frac{\sin 4x}{x}=$

You must know these limits! MEMORIZE!

1. 2. 3.

Practice: $\lim_{x\to -1}\frac{2x^2-x-3}{x+1}$ $\lim_{x\to 0}\frac{\sin(3x)}{x}$

$\lim_{x\to 3}\frac{x^2-x-6}{x+3}$ $\lim_{x\to 0}\frac{\cos x\tan x}{x}$

$\lim_{x\to -1}\frac{x^2-1}{(x+1)^2}$ $\lim_{x\to 2}\frac{x-\sqrt{6-x}}{(x-2)}$